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Impact of Effective Stress and Matrix Deformation on the Coal Fracture Permeability

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Abstract The permeability of coal is an important parameter in mine methane control and coal bed methane exploitation because it determines the practicability of methane extraction. We developed a new coal permeability model under tri-axial stress conditions. In our model, the coal matrix is compressible and Biot's coefficient, which is considered to be 1 in existing models, varies between 0 and 1. Only a portion of the matrix deformation, which is represented by the effective coal matrix deformation factor f_m, contributes to fracture deformation. The factor f_m is a parameter of the coal structure and is a constant between 0 and 1 for a specific coal. Laboratory tests indicate that the Sulcis coal sample has an f_m value of 0.1794 for N_2 and CO_2. The proposed permeability model was evaluated using published data for the Sulcis coal sample and is compared to three popular permeability models. The proposed model agrees well with the observed permeability changes and can predict the permeability of coal better than the other models. The sensitivity of the new model to changes in the physical, mechanical and adsorption deformation parameters of the coal was investigated. Biot's coefficient and the bulk modulus mainly affect the effective stress term in the proposed model. The sorption deformation parameters and the factor f_m affect the coal matrix deformation term.

Keywords Coal permeability · Coal matrix · Effective deformation · Biot's coefficient

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List of Symbols

ε_e	Coal bulk strain caused by the effective stress (dimensionless)
ε_eP	Fracture strain caused by the effective stress (dimensionless)
V_e	Coal bulk volume caused by the effective stress (mL)
V_ef	Fracture volume caused by the effective stress (mL)
V	Coal bulk volume (mL)
V_P	Coal pore volume (mL)
$\bar{\sigma}$	Mean stress (MPa)
K	Coal bulk modulus (MPa)
K_P	Coal pore system modulus (MPa)
α	Biot's coefficient (dimensionless)
K_m	Coal matrix modulus (MPa)
β	Effective coefficient of fracture (dimensionless)
ϕ	Fracture porosity of coal (dimensionless)
ε_m	Coal matrix strain (dimensionless)
ε_s	Coal matrix strain due to sorption (dimensionless)
ε_mP	Coal matrix strain due to gas pressure compression (dimensionless)
ε_max	Maximum adsorption strain (dimensionless)
P	Gas pressure (MPa)
P_L	Langmuir's pressure (MPa)
P_R	Rebound pressure (MPa)
V_mf	Fracture volume deformation due to coal matrix deformation (mL)
V_mv	Bulk volume deformation due to matrix deformation (mL)
V_m	Coal matrix volume (mL)
k	Coal permeability (mD)
E	Elastic modulus (MPa)
υ	Poisson's ratio (dimensionless)
M	Constrained axial modulus (MPa)
f_m	Effective coal matrix deformation factor (dimensionless)
f	Empirical parameter for P–M model (dimensionless)
γ	Matrix compressibility (MPa^{-1})
C_f	Fracture compressibility (MPa^{-1})
C_0	Initial fracture compressibility (MPa^{-1})
θ	Decline rate of fracture compressibility with increasing effective stress (MPa^{-1})

Subscript

0	Initial or reference state

1 Introduction

Coal bed methane (CBM) is a natural product of the coalification process (Yu 1992; Zhou and Lin 1997). CBM is a serious threat to safety in underground coal mining and can cause disasters, such as coal and gas outbursts and gas explosions (Yu 1992; Karacan et al. 2011). However, CBM is also an unconventional natural gas resource that has been exploited worldwide in such countries as in the USA, Australia and China (Liu et al. 2011; Moore 2012).

Coal permeability is an important parameter in mine methane control and CBM exploitation, because it determines the practicability of methane extraction. The permeability of coal depends on the fracture characteristics, including the size, spacing, connectivity, width, mineral fill and distribution (Laubach et al. 1998). CBM extraction causes a series of coal-gas interactions. The decrease in CBM pressure caused by extraction leads to an increase in the effective stress. As a result, the closing of fractures causes the coal permeability to decrease. At the same time, the adsorbed CBM desorbs from the coal matrix due to the decreased pressure, which leads to shrinkage of the coal matrix. The opening of the fractures because of matrix shrinkage increases the coal permeability. The increase or decrease of the coal permeability depends on the net effect of the processes described above (Connell and Detournay 2009).

Several models have been proposed to explain the variability of coal permeability. Coal permeability models can be divided into two important classes: those under uniaxial strain conditions and those under tri-axial stress conditions (Liu et al. 2011). Among them, permeability models under uniaxial strain conditions were established by Gray (1987), Sawyer et al. (1990), Seidle and Huitt (1995), Palmer and Mansoori (1998), Shi and Durucan (2004), Cui and Bustin (2005). Robertson and Christiansen (2006), Zhang et al. (2008), Liu and Rutqvist (2010), Connell et al. (2010a) and Liu et al. (2010) proposed permeability models under tri-axial stress conditions.

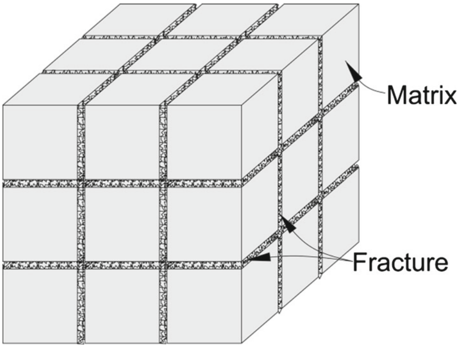
However, uniaxial strain conditions are a simplified homogenisation of the stress–strain states of coal during mining and exploitation and may be valid at the scale of a relatively large basin; the mechanical conditions at the local scale are expected to be much more complex in coal seams (Liu and Rutqvist 2010). And laboratory permeability tests are conducted under tri-axial stress–strain conditions (Robertson and Christiansen 2006; Zhang et al. 2008; Liu and Rutqvist 2010; Connell et al. 2010a; Liu et al. 2010). Therefore, a coal permeability model under conditions of tri-axial stress–strain can be used to investigate the influence of factors and the variability of coal permeability more comprehensively than other models.

The effect of effective stress was considered by most models except that of Seidle and Huitt (1995) who assumed that cleat deformation was caused entirely by desorption shrinkage. The coal matrix is assumed to be incompressible by assuming that the bulk modulus of the coal matrix is much larger than the coal bulk modulus, and then Biot's coefficient α is assumed to be 1 (Gray 1987; Sawyer et al. 1990; Seidle and Huitt 1995; Palmer and Mansoori 1998; Shi and Durucan 2004; Cui and Bustin 2005; Liu and Rutqvist 2010; Connell et al. 2010a). However, the compression of the coal matrix by the pore pressure could not been ignored (Pan and Connell 2007; Hol and Spiers 2012). Therefore, the Biot's coefficient for coal is less than 1 (Durucan et al. 2009; Connell et al. 2010b; St. George and Barakat 2001).

Most models consider the matrix deformation to be equal to the fracture deformation. However, only part of the matrix deformation contributes to the fracture deformation (Robertson and Christiansen 2005). Connell et al. (2010a) and Liu and Rutqvist (2010) established permeability models in which the sorption deformation partly applied to the fracture.

In situ coal is subjected to complex stress–strain conditions, and the variability of the coal permeability is controlled by the stress, the gas pressure and the nature of the coal. The primary objective of this study is to develop a new coal permeability model that considers the effect of effective stress on the fracture deformation and also takes into account the partial contributions of coal matrix deformation that is caused by sorption under tri-axial conditions. The sensitivity of the new model to changes in the physical, mechanical and adsorption deformation parameters of coal will be investigated as well.

Fig. 1 Dual porosity model of coal (Warren and Root 1963)



2 Establishment of the Permeability Model

The coal has a natural dual porosity structure that consists of the coal matrix and the fracture in which there are numerous inorganic minerals, mainly kaolinite, pyrite and illite, as shown in Fig. 1. More than 95 % of the gas occurs as adsorbed gas in the sorption space of the abundant micro-pores (Gray 1987). The gas migrates by diffusion in the micro-pore system and follows Fick's Law. The closely spaced natural fractures surrounding the coal matrix, which form the cleat system, determine the mechanical properties of the coal and the flow paths for the methane; this flow follows Darcy's Law. Therefore, the coal fracture permeability is closely related to the characteristics of the fractures, which are controlled by the coal rank, formation stress, geologic structure, mining and other factors. During mining or exploitation, the coal fractures are dominantly affected by the coal mining stress and the gas pressure. Below, we analyse the contributions of stress, gas pressure and sorption on the fracture deformation by dividing the effect of sorptive gas into the effect of effective stress and the effect of sorption deformation of the coal matrix.

2.1 Basic Assumptions

The following assumptions are made to simplify the model:

(1) Coal is considered to be a dual continuous isotropic elastic medium even though the coal consists of the coal matrix and fracture. We abstract the fracture (cleat) system as a pore system and use the poroelastic theory to analyse the fracture (cleat) deformation (Maghous et al. 2013). The porosity is the fracture (cleat) porosity henceforth.

(2) The strain is elastic and infinitesimal, so the second and higher order terms can be ignored. Therefore, the strains induced by the different factors can be added.

2.2 Effective Stress

As a porous medium, the coal bulk volume V is composed of the matrix volume V_m and the pore volume V_P

$$V = V_\mathrm{P} + V_\mathrm{m}. \tag{1}$$

According to the effective stress principle (Biot 1941), the bulk volumetric strain increment can be expressed as

$$\mathrm{d}\varepsilon_\mathrm{e} = \frac{\mathrm{d}V_\mathrm{e}}{V} = -\frac{1}{K}\left(\mathrm{d}\bar{\sigma} - \alpha \mathrm{d}P\right) \tag{2}$$

and the pore volume strain increment can be expressed similarly

$$\mathrm{d}\varepsilon_{\mathrm{eP}} = \frac{\mathrm{d}V_{\mathrm{eP}}}{V_{\mathrm{P}}} = -\frac{1}{K_{\mathrm{P}}}\left(\mathrm{d}\bar{\sigma} - \beta \mathrm{d}P\right) \tag{3}$$

where ε_{e} and V_{e} are the coal bulk strain and volume caused by the effective stress, respectively; $\varepsilon_{\mathrm{eP}}$ and V_{eP} are the pore strain and volume caused by the effective stress, respectively; $\bar{\sigma} = \frac{1}{3}(\sigma_{11}+\sigma_{22}+\sigma_{33})$ is the mean stress, MPa; $K = \frac{E}{3(1-2v)}$ is the coal bulk modulus, MPa; K_{P} is the coal pore modulus, MPa; $\alpha = 1 - K/K_{\mathrm{m}}$ is Biot's coefficient; $\beta = 1 - K_{\mathrm{P}}/K_{\mathrm{m}}$ is the effective coefficient for the pore system; K_{m} is the coal matrix modulus, MPa; E is the elastic modulus of the coal, MPa and v is Poisson's ratio.

Without the gas sorption effect, the volumetric change of the porous medium satisfies the Betti–Maxwell reciprocal theorem (Detournay and Cheng 1993), $(\partial V/\partial P)_{\bar{\sigma}} = (\partial V_{\mathrm{P}}/\partial \bar{\sigma})_{\mathrm{P}}$, and we obtain

$$K_{\mathrm{P}} = \frac{\phi}{\alpha} K \tag{4}$$

where ϕ is the fracture porosity.

2.3 The Coal Matrix Deformation

The coal matrix swells in the presence of the sorptive gas and is simultaneously compressed by the gas pressure. The deformation of the coal matrix is the result of the net difference between the two effects (Pan and Connell 2007; Hol and Spiers 2012; St. George and Barakat 2001). Therefore, the sorption strain must be calibrated by deducting the gas compression from experimental data, that is

$$\mathrm{d}\varepsilon_{\mathrm{s}} = \mathrm{d}\varepsilon_{\mathrm{exp}} - \frac{\mathrm{d}P}{K_{\mathrm{m}}} \tag{5a}$$

$$\varepsilon_{\mathrm{s}} = \varepsilon_{\mathrm{exp}} - \frac{P}{K_{\mathrm{m}}} \tag{5b}$$

where ε_{s} is the coal sorption strain; $\varepsilon_{\mathrm{exp}}$ is the experimental strain measured directly and P is the gas pressure, MPa.

(1) Coal matrix sorption strain

The coal matrix can swell when it adsorbs methane and other sorptive gases. The strain can be described using an equation in Langmuir's form

$$\varepsilon_{\mathrm{s}} = \frac{\varepsilon_{\max} P}{P + P_{\mathrm{L}}} \tag{6}$$

where $\varepsilon_{\max}$ is the maximum adsorption strain when the gas pressure is infinite; P_{L} is the pressure when the adsorption strain is half of the maximum adsorption strain, which is called the Langmuir pressure, MPa.

(2) Fracture deformation caused by the coal matrix deformation

Deformation of the coal matrix can affect the deformation of both the bulk coal and the fractures in the coal (Robertson and Christiansen 2006; Cui et al. 2007; Seidle and Huitt 1995; Palmer and Mansoori 1998). The coal matrix deformation is assumed to contribute entirely to the fracture deformation (Palmer and Mansoori 1998; Seidle and Huitt 1995; Robertson and Christiansen 2006; Zhang et al. 2008). However, the contribution of coal matrix deformation to the fractures has been significantly overestimated (Robertson and Christiansen 2005; Liu and Rutqvist 2010; Connell et al. 2010a). For example, Robertson and Christiansen (2005) demonstrated that the most commonly used models (Palmer and Mansoori 1998; Shi and Durucan 2004) significantly overestimate the effects of matrix swelling on the permeability changes observed in laboratory experiments.

Pone et al. (2009) analysed the various types of deformation in coal samples that adsorb CO_2 under confining stress using high-resolution X-ray CT technology. Their results show that the fracture aperture decreases partly due to the swelling of the adjacent coal matrix. Numerous inorganic minerals, mainly kaolinite, pyrite and illite, are present in coal fractures (Karacan 2007; Dawson et al. 2012), and these minerals prevent the coal matrix from completely closing the fracture. Therefore, only part of the matrix deformation contributes to the fracture deformation. When adsorbing the gas, the inner parts of the coal can automatically adjust to the deformation (Karacan 2003, 2007). The effective coal matrix deformation factor, f_{m}, is introduced to measure the degree of influence of the coal matrix deformation on the fracture deformation. The factor f_{m} is a parameter of the coal structure and depends on the distribution of fractures, the characteristics of the fracture fill and other factors.

The parameter f_{m} may be a complex function of the fracture characteristics and others. For a first approximation, we assume f_{m} is a constant which is applicable. Therefore, f_{m} is a constant between 0 and 1 for a particular coal. If there is no fracture in the coal, the parameter $\mathrm{f_m}$ is equal to 0. The parameter f_{m} would be equal to 1 when two surfaces of the fracture are smooth and parallel.

Thus, the fracture deformation due to the coal matrix deformation is expressed as

$$\mathrm{d}V_{\mathrm{mf}} = f_{\mathrm{m}}\mathrm{d}V_{\mathrm{m}} = f_{\mathrm{m}}V_{\mathrm{m}}\mathrm{d}\varepsilon_{\mathrm{s}}. \tag{7}$$

where V_{mf} is the fracture volume deformation due to deformation of the coal matrix; and V_{m} is the volume of the coal matrix.

2.4 The Permeability Model Under Tri-axial Stress Conditions

Based on the definition of porosity, $\phi = V_{\mathrm{P}}/V$, we obtain

$$\mathrm{d}\phi = \mathrm{d}\left(\frac{V_{\mathrm{P}}}{V}\right) = \phi\left(\frac{\mathrm{d}V_{\mathrm{P}}}{V_{\mathrm{P}}} - \frac{\mathrm{d}V}{V}\right). \tag{8}$$

The bulk volume deformation of the coal is equal to the sum of the deformation due to the effective stress and the coal matrix deformation due to adsorption and gas pressure compression

$$\mathrm{d}V = \mathrm{d}V_{\mathrm{e}} + \mathrm{d}V_{\mathrm{mv}} = -\frac{V}{K}\left(\mathrm{d}\bar{\sigma} - \alpha\mathrm{d}P\right) + \left(1 - f_{\mathrm{m}}\right)V_{\mathrm{m}}\mathrm{d}\varepsilon_{\mathrm{s}} \tag{9}$$

where V_{mv} is the bulk volume deformation due to the matrix deformation.

Dividing both sides of Eq. (9) by the coal bulk volume, we obtain

$$\frac{\mathrm{d}V}{V} = -\frac{1}{K}\left(\mathrm{d}\bar{\sigma} - \alpha\mathrm{d}P\right) + \left(1 - f_{\mathrm{m}}\right)\left(1 - \phi\right)\mathrm{d}\varepsilon_{\mathrm{s}}. \tag{10}$$

Similarly, from Eqs. (3) and (7) we obtain

$$\frac{\mathrm{d}V_{\mathrm{P}}}{V_{\mathrm{P}}} = -\frac{1}{K_{\mathrm{P}}}\left(\mathrm{d}\bar{\sigma} - \beta\mathrm{d}P\right) - \frac{1-\phi}{\phi}f_{\mathrm{m}}\mathrm{d}\varepsilon_{\mathrm{s}}. \tag{11}$$

By substituting Eqs. (10) and (11) into Eq. (8), we obtain

$$\frac{\mathrm{d}\phi}{\phi} = -\frac{1}{K_\mathrm{P}}\left(\mathrm{d}\bar{\sigma} - \beta \mathrm{d}P\right) + \frac{1}{K}\left(\mathrm{d}\bar{\sigma} - \alpha \mathrm{d}P\right) - \left[\frac{1-\phi}{\phi} f_\mathrm{m} + (1 - f_\mathrm{m})(1-\phi)\right]\mathrm{d}\varepsilon_\mathrm{s}. \tag{12}$$

Then, substituting $K_\mathrm{P} = \frac{\phi}{\alpha}K$ and $\beta = 1 - K_\mathrm{P}/K_\mathrm{m}$ into Eq. (12) and considering that $\phi \ll 1(\phi < 10\,\%)$, we can rearrange and simplify the equation to obtain

$$\mathrm{d}\phi = -\frac{\alpha}{K}\left(\mathrm{d}\bar{\sigma} - \mathrm{d}P\right) - f_\mathrm{m}\left(\mathrm{d}\varepsilon_\mathrm{s} - \mathrm{d}\varepsilon_\mathrm{mP}\right). \tag{13}$$

Integrating Eq. (13) gives

$$\phi = \phi_0 - \frac{\alpha}{K}\left[(\bar{\sigma} - \bar{\sigma}_0) - (P - P_0)\right] - f_\mathrm{m}\left(\frac{\varepsilon_\mathrm{max} P}{P + P_\mathrm{L}} - \frac{\varepsilon_\mathrm{max} P_0}{P_0 + P_\mathrm{L}}\right). \tag{14}$$

The widely used cubic relationship between permeability and porosity (Gray 1987; Sawyer et al. 1990; Seidle and Huitt 1995; Palmer and Mansoori 1998; Shi and Durucan 2004; Robertson and Christiansen 2006; Zhang et al. 2008; Liu and Rutqvist 2010; Connell et al. 2010a) is given as

$$\frac{k}{k_0} = \left(\frac{\phi}{\phi_0}\right)^3 \tag{15}$$

where k is the coal permeability.

Substituting Eq. (14) into (15), the coal permeability model that considers the effect of the effective stress and coal matrix deformation (ESMD model) is given as

$$\frac{k}{k_0} = \left\{1 - \underbrace{\frac{\alpha}{\phi_0 K}\left[(\bar{\sigma} - \bar{\sigma}_0) - (P - P_0)\right]}_{\text{Effect of effective stress}} - \underbrace{\frac{f_\mathrm{m}}{\phi_0}\left(\frac{\varepsilon_\mathrm{max} P}{P + P_\mathrm{L}} - \frac{\varepsilon_\mathrm{max} P_0}{P_0 + P_\mathrm{L}}\right)}_{\text{Effect of coal matrix deformation}}\right\}^3. \tag{16}$$

It is clear that the model contains an effective stress term and a coal matrix deformation term. The factor f_m measures the degree of influence of the coal matrix deformation on the fracture deformation.

2.5 Rebound Pressure

Laboratory tests on coal permeability are usually carried out under hydrostatic conditions. Thus, we have calculated the rebound pressure P_R at which the permeability changes from a decrease to an increase by taking the derivative of Eq. (16) with respect to the gas pressure P under hydrostatic conditions of constant stress and varying pressure. The rebound pressure is expressed as

$$P_\mathrm{R} = \sqrt{\frac{f_\mathrm{m}\varepsilon_\mathrm{max} P_\mathrm{L} K}{\alpha}} - P_\mathrm{L}. \tag{17}$$

If $P_\mathrm{R} > 0$, the permeability change will reverse at the rebound pressure P_R. Otherwise, the permeability increases throughout with gas pressure increase under constant stress conditions.

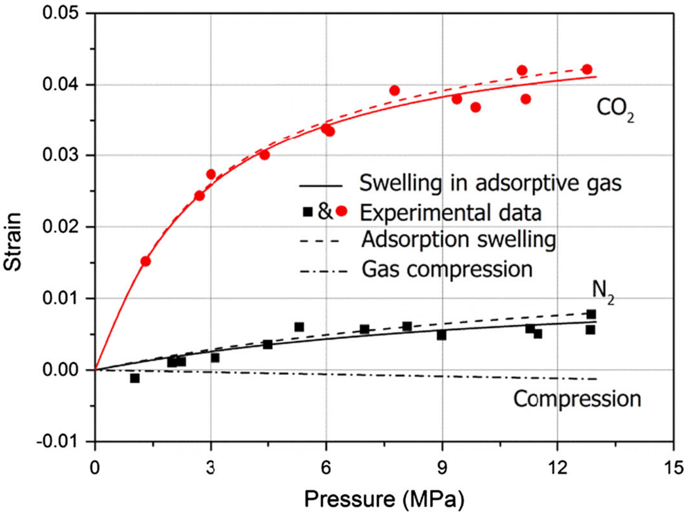


Fig. 2 The coal matrix swelling in adsorptive gas (experimental data from Pini et al. 2009)

3 Model Validation and Evaluation

3.1 Experimental Data

Numerous laboratory experiments have been conducted on coal permeability (Chen et al. 2011; Robertson and Christiansen 2005; Pini et al. 2009). Pini et al. (2009) conducted experiments that tested the mechanical parameters, porosity, adsorption swelling parameters and coal permeability of a coal sample (Sulcis coal sample) from the Monte Sinni coal mine in the Sulcis Coal Province (Sardinia, Italy). We use the experimental data to validate and evaluate the ESMD model because of the comprehensive set of parameters available for the coal sample and the detailed experimental data. The coal permeability experiments were conducted under hydrostatic conditions at a constant confining stress (10 MPa) and various gas pressures between 0 and 8 MPa at 45 °C using N_2 and CO_2.

Substituting Eq. (6) into Eq. (5b), we could rearrange the equation to obtain

$$\varepsilon_{\text{exp}} = \frac{\varepsilon_{\max} P}{P + P_{\text{L}}} - \frac{P}{K_{\text{m}}} \tag{18}$$

Thus, the adsorption swelling parameters of the Sulcis coal sample for N_2 and CO_2 were corrected using Eq. (18) as shown in Fig. 2. The parameters of the Sulcis coal sample are shown in Table 1.

3.2 Validation

The experimental data are matched by the ESMD model using the parameters in Table 1. The results are shown in Fig. 3. The ESMD model can match the experimental data well. As shown in Fig. 3, the experimental data for CO_2 and the ESMD model prediction indicate that the coal permeability decreases as the pressure increases at lower pressures due primarily to swelling of the coal matrix during sorption. With a further increase in gas pressure, the effective stress gradually plays a greater role, and the coal permeability increases due to the

Table 1 Parameters and magnitudes (Pini et al. 2009)

Parameter	Value
Elastic modulus, E (MPa)	1119
Poisson's ratio, ν	0.26
Bulk modulus, K (MPa)	778
Matrix modulus, K_m (MPa)	10,340
Constrained axial modulus, M (MPa)	1,369
Boit's coefficient, α	0.925
Initial porosity (for N_2), ϕ_0 (%)	0.5834
Initial porosity (for CO_2), ϕ_0 (%)	0.42
Maximum sorption strain (for N_2), ε_{max}	0.017
Langmuir pressure (for N_2), P_L (MPa)	14.72
Maximum sorption strain (for CO_2), ε_{max}	0.05187
Langmuir pressure (for CO_2), P_L (MPa)	2.913
Effective coal matrix deformation factor, f_m	0.1794
Empirical parameter for P–M model, f	0.1
Matrix compressibility, γ (MPa^{-1})	9.67E−05
Fracture compressibility, C_f (MPa^{-1})	0.013
Initial fracture compressibility, C_0 (MPa^{-1})	0.3422
Decline rate of fracture compressibility with increasing effective stress, θ (MPa^{-1})	2.65E−14

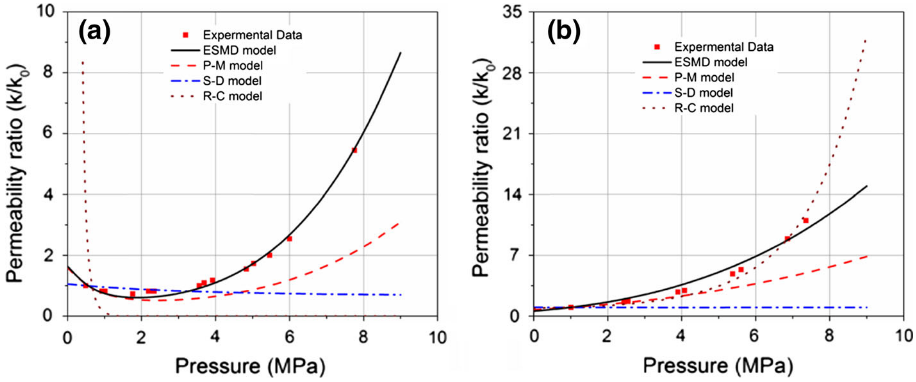


Fig. 3 Model results compared to experimental data: **a** for CO_2 and **b** for N_2. The confining pressure is 10 MPa, and the temperature is 45 °C

decreasing effective stress caused by the increasing gas pressure. For N_2, the coal permeability increases gradually across the range of increasing pressure because the matrix adsorption swelling capacity of the Sulcis coal sample is low for N_2; this makes the effective stress to play a dominant role, which can be interpreted from Eq. (17). For CO_2, P_R=1.87 MPa, which indicates that the permeability rebounds at a gas pressure of 1.87 MPa. However, $P_R = -8.58$ MPa for N_2, which implies that the permeability increases throughout with increasing gas pressure.

The factor f_m of the Sulcis coal sample is 0.1794, which was obtained by matching the experimental data for N_2 and CO_2. This verifies that f_m is a structural parameter of coal.

Table 2 Widely used permeability models

Models	Formula description
P–M model (Palmer and Mansoori 1998)	$\frac{k}{k_0}=\left[1+\frac{C_\mathrm{m}}{\phi_0}(P-P_0)+\frac{\varepsilon_\mathrm{max}}{3\phi_0}\left(\frac{K}{M}-1\right)\left(\frac{P}{P_\mathrm{L}+P}-\frac{P_0}{P_\mathrm{L}+P_0}\right)\right]^3$ $C_\mathrm{m}=\frac{1}{M}-\left(\frac{K}{M}+f-1\right)\gamma,\ M=\frac{E(1-v)}{(1+v)(1-2v)}$
S–D model (Shi and Durucan 2004)	$\frac{k}{k_0}=\exp\left\{3C_f\left[\frac{v}{1-v}(P-P_0)-\frac{\varepsilon_\mathrm{max}}{3}\frac{E}{1-v}\left(\frac{P}{P_\mathrm{L}+P}-\frac{P_0}{P_\mathrm{L}+P_0}\right)\right]\right\}$
R–C model (Robertson and Christiansen 2006)	$\frac{k}{k_0}=\exp\left\{3C_0\frac{1-\exp[\theta(P-P_0)]}{-\theta}\right.$ $\left.+\frac{9}{\phi_0}\left[\frac{1-2v}{E}(P-P_0)-\frac{\varepsilon_\mathrm{max}P_\mathrm{L}}{(P_\mathrm{L}+P_0)}\ln\left(\frac{P_\mathrm{L}+P}{P_\mathrm{L}+P_0}\right)\right]\right\}$

Table 3 Comparison of permeability factors between the models

Permeability models	Biot's coefficient	Sorption inducing matrix deformation	Effective coal matrix deformation factor
P–M model	1	Yes	1
S–D model	1	Yes	1
R–C model	1	Yes	1
ESMD model	0–1	Yes	0–1

3.3 Evaluation

Many coal permeability models have been developed, and we compare the ESMD model to three of the most popular models: the Palmer–Mansoori (P–M) model (Palmer and Mansoori 1998), the Shi–Durucan (S–D) model (Shi and Durucan 2004) and the Robertson-Christiansen (R–C) model (Robertson and Christiansen 2006) using the experimental data from Pini et al. (2009). The three models are shown in Table 2. The models are compared in Fig. 3.

The three models (P–M, S–D and R–C) were matched to the experimental data (Pini et al. 2009) using the parameters in Table 1. In the P–M model, parameter f is obtained by fitting. The matrix compressibility γ is the reciprocal of the coal matrix modulus. In the S–D model, the fracture compressibility C_f is obtained by fitting. In the R–C model, the initial fracture compressibility C_0 and the decline rate of the fracture compressibility with increasing effective stress θ are also obtained by matching. These values are shown in Table 1.

The three models poorly match the experimental data for two reasons: in all three models, Biot's coefficient is assumed to be 1 in the P–M model and the S–D model by assuming that the coal matrix is incompressible, as shown in Table 3. Deformation of the coal matrix contributes to the fracture deformation entirely in the three models, which is an overestimation. As shown in Fig. 3, the R–C model matches the experimental data well for N_2 but poorly for CO_2. The decline rate of fracture compressibility with increasing effective stress θ for the R–C model is 2.65E−14 MPa^{-1}, which implies that the fracture compressibility does not vary with the effective stress. However, the decline rate θ varies between 2.45E−2 and 2.61E−1 MPa^{-1} (Robertson and Christiansen 2005, 2006; McKee et al. 1988).

Only part of the matrix deformation caused by sorption contributes to the fracture deformation. The factor f_m, which ranges from 0 to 1, is introduced to measure the degree of

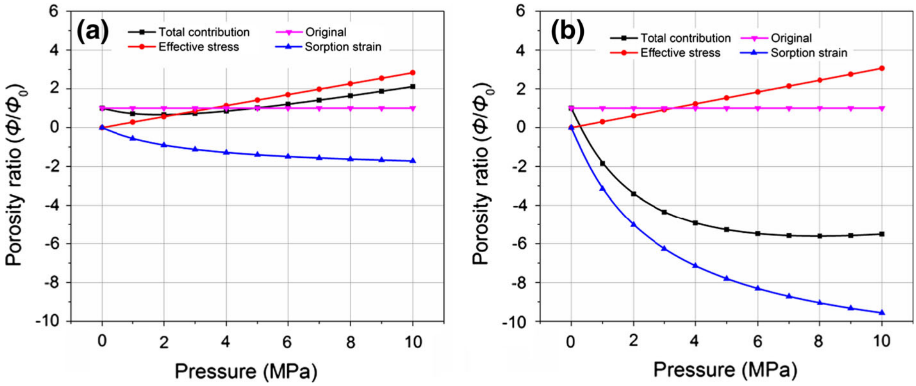


Fig. 4 The relative contributions of model terms (**a** computation based on the parameters in Table 1; **b** computation based on the parameters in Table 1 with $\alpha = 1$ and $f_\mathrm{m} = 1$)

influence of the coal matrix deformation on the fracture deformation in the ESMD model. The factor f_m is a parameter of the coal structure and does not vary with the type of gas. The factor f_m for the Sulcis coal sample is 0.1794 for both N_2 and CO_2. Biot's coefficient α of coal is less than 1, and $\alpha = 0.925$ for the Sulcis coal sample, which has a bulk modulus of 778 MPa and a matrix modulus of 10,340 MPa. Therefore, the ESMD model matches the experimental data of the Sulcis coal sample well for both N_2 and CO_2.

3.4 Contribution of Terms in the Permeability Model

As shown in Fig. 4, the contribution of the terms in Eq. (16) to the porosity variation has been calculated for the CO_2 permeability experiment of the Sulcis coal sample. At lower pressures, the coal matrix sorption deformation plays a dominant role, and the coal porosity decreases. With an increase of gas pressure, the effective stress gradually plays a larger role, and the coal porosity increases due to the decrease in effective stress caused by the increase of gas pressure.

We calculated the contribution of the items in Eq. (16) to the porosity variation for the CO_2 permeability experiment of the Sulcis coal sample assuming that $\alpha = 1$ and $f_\mathrm{m} = 1$. Although the contribution of the effective stress is overestimated, the contribution of the coal matrix sorption deformation is overestimated more severely, as shown in Fig. 4b. The coal matrix sorption deformation plays a dominant role through the entire process. The coal porosity decreases with increasing pressure. But the porosity of coal decreases below zero with gas pressure increase, which is unrealistic.

4 Sensitivity of the ESMD Model to the Input Parameters

We have discussed the sensitivity of the ESMD model to the input parameters, such as the coal bulk modulus K, the adsorption swelling deformation parameters ε_max and P_L, Biot's coefficient α and the effective coal matrix deformation factor f_m, assuming that the coal is under hydrostatic conditions of constant confining stress (10 MPa) and various gas pressures between 0 and 8 MPa at constant temperature. The magnitudes of the parameters used in the calculations are shown in Table 4.

Table 4 Magnitudes of parameters used in the calculations

Parameter	Value
Bulk modulus, K (MPa)	1200
Maximum sorption strain, $\varepsilon_{\max}$	0.02
Langmuir pressure, P_L (MPa)	3
Initial porosity, ϕ_0	0.005
Biot's coefficient, α	0.8
Effective coal matrix deformation factor, f_m	0.2

Table 5 Coal characterisation data obtained from Durucan et al. (2009)

Parameter	Value						
	Schwalbach	W–L no.1	Splint	Tupton	Dora	Selar 9 ft	Tower 7 ft
Vitrinite reflectance, %	0.79	0.71	0.55	0.49	0.71	2.41	2.28
Elastic modulus, E (GPa)	3.55	2.44	2.05	1.36	2.63	2.165	2.04
Poisson's ratio, υ	0.26	0.42	0.34	0.36	0.38	0.4	0.32
Matrix compressibility, γ (MPa^{-1})	21.75E−6	48.10E−6	27.55E−6	47.85E−6	65.00E−6	40.10E−6	41.30E−6
Bulk modulus[a], K (GPa)	2.47	5.08	2.14	1.62	3.65	3.61	1.89
Matrix modulus[a], K_m (GPa)	45.98	20.79	36.30	20.90	15.38	24.94	24.21
Biot's coefficient[a], α	0.95	0.76	0.94	0.92	0.76	0.86	0.92

[a] Bulk modulus K, matrix modulus K_m and Biot's coefficient α are calculated using the data from Durucan et al. (2009), where $K = \frac{E}{3(1-2\upsilon)}$, $K_m = \frac{1}{\gamma}$ and $\alpha = 1 - K/K_m$.

4.1 Biot's Coefficient

Biot's coefficient α indicates the difference between the coal bulk modulus and the matrix modulus. The smaller the value of α is, the closer the coal bulk modulus is to the matrix modulus and vice versa. It has been verified that Biot's coefficient α of coal is less than 1. Durucan et al. (2009) determined the mechanical parameters of the various ranks of European Coal. Biot's coefficient was calculated to range from 0.76 to 0.95 with an average of 0.87 (Table 5).

We calculated the variation in coal permeability for a range of Biot's coefficient from 0.7 to 1. As shown in Fig. 5, the permeability decreases with an initial increase in gas pressure and increases gradually with a continued increase in pressure. The smaller Biot's coefficient is, the more the coal permeability decreases before the rebound pressure and the less the coal permeability increases after the rebound pressure. The rebound pressure increases with a decrease in Biot's coefficient.

Biot's coefficient appears in the mechanical term of Eq. (16), that is the effective stress term. The bigger the Biot's coefficient is, the greater the contribution of effective stress to the coal permeability is.

4.2 Coal Bulk Modulus

The coal bulk modulus reflects the ability of coal to resist deformation. The greater the coal bulk modulus, the stronger the ability to resist deformation. We calculated the variation in

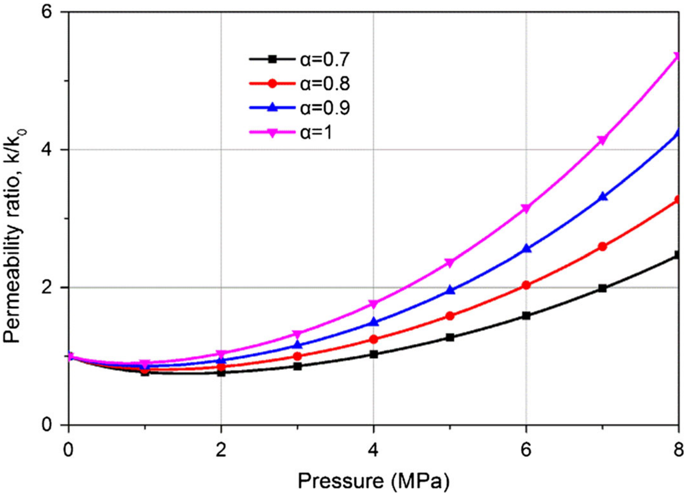


Fig. 5 Sensitivity of the ESMD model to changes in Biot's coefficient of coal

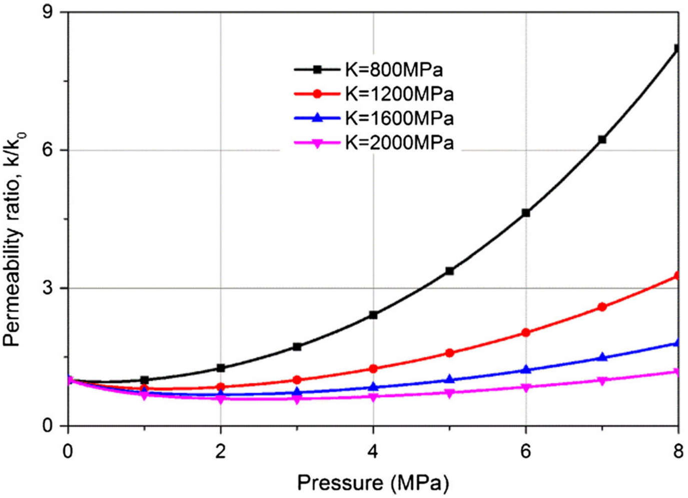


Fig. 6 Sensitivity of the ESMD model to changes in the bulk modulus of coal

coal permeability for a range in coal bulk modulus from 800 to 2,000 MPa (Fig. 6). Using the parameters in Table 4, the rebound pressure increases from 0.46 MPa at $K = 800$ to 2.48 MPa at $K = 2,000$ MPa. The rebound pressure increases with increasing bulk modulus. The permeability decreases at pressures lower than the rebound pressure and then increases gradually above the rebound pressure with increasing gas pressure. The higher the bulk modulus is, the more the coal permeability decreases below the rebound pressure and the less the coal permeability increases above rebound pressure.

The coal bulk modulus also appears in the mechanical term of Eq. (16). The increase of bulk modulus weakens the effect of the mechanical term on the permeability. Therefore, the phenomenon described in the previous paragraph occurs when the coal bulk modulus increases.

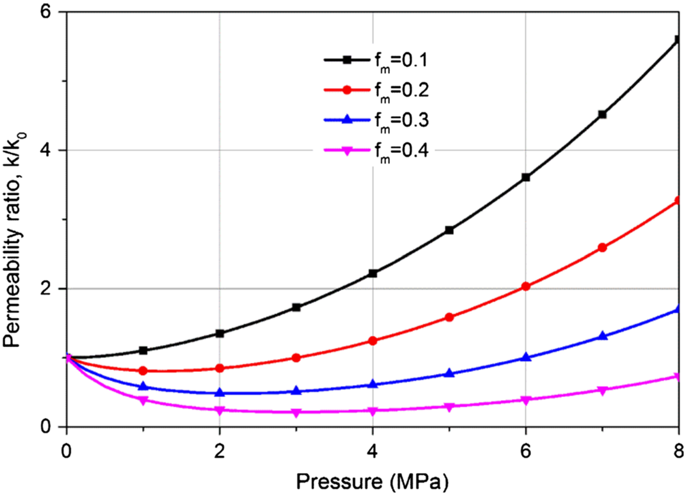


Fig. 7 Sensitivity of the ESMD model to changes in f_m of coal

4.3 The Effective Coal Matrix Deformation factor

The effective coal matrix deformation factor f_m is introduced to measure the degree of influence of the coal matrix deformation on the fracture deformation. The factor f_m is a parameter of the coal structure and depends mainly on the distribution of fractures, the fracture fill characteristics and other factors. For a particular coal, f_m is a constant between 0 and 1. The factor f_m of the Sulcis coal sample is 0.1794 and was obtained by matching the experimental data for N_2 and CO_2. This further verifies that f_m is a structural parameter of coal. The factor f_m may be obtained by determining the sorption deformation of the bulk coal and the coal matrix, but additional studies are required.

We calculated the variation in coal permeability for a range of f_m from 0.1 to 0.4 (Fig. 7). The rebound pressure increases from 0 to 3 MPa with an increase of f_m from 0.1 to 0.4. The rebound pressure of 0 MPa at f_m=0.1 indicates that the permeability increases with increasing gas pressure. When the rebound pressure is greater than 0, the higher f_m is, the more the coal permeability decreases below the rebound pressure and the less the coal permeability increases above the rebound pressure.

The factor f_m appears in the coal matrix deformation term of Eq. (16). Larger values of f_m enhance the effect of coal matrix deformation. Therefore, the phenomenon described in the previous paragraph occurs when f_m increases.

4.4 Sorption Deformation

The coal matrix swells when adsorbing gas, which is described by the sorption deformation ε_{max} and P_L. We calculated the variation in coal permeability for a range of ε_{max} from 0.005 to 0.03, as shown in Fig. 8a, and a range of P_L from 1 to 9 MPa, as shown in Fig. 8b.

The rebound pressure increases from −0.88 to 2.20 MPa with an increase in ε_{max} from 0.005 to 0.03. The rebound pressure of −0.88 MPa at $\varepsilon_{max} = 0.005$ indicates that the permeability increases with increasing gas pressure. The changes in coal permeability and rebound pressure with increasing ε_{max} are similar to that with the increase in f_m.

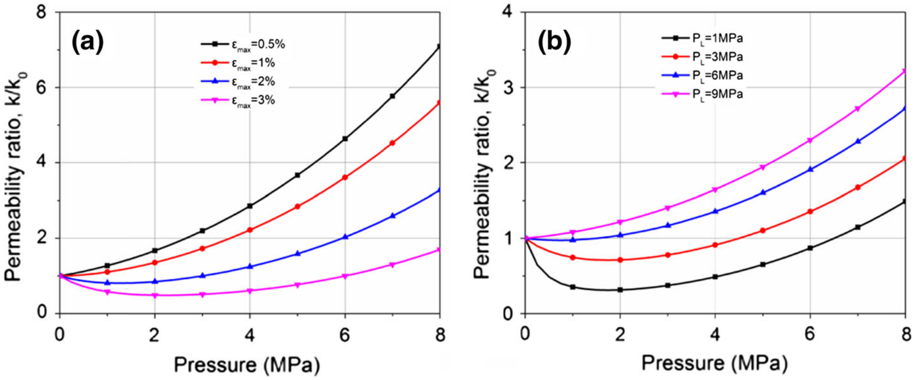


Fig. 8 Sensitivity of the ESMD model to changes in the sorptive-elastic properties of coal $\varepsilon_{\max}$ and P_{L}

However, the rebound pressure appears to increase first and then decreases with increasing Langmuir pressure as shown in Fig. 8b. We calculated the Langmuir pressure at which the rebound pressure changes from increasing to decreasing by taking the derivative of Eq. (17) with respect to P_{L} and letting the derivative be equal to zero. The Langmuir pressure is expressed as

$$P_{\mathrm{L}} = \frac{1}{4}\frac{f_{\mathrm{m}} K \varepsilon_{\max}}{\alpha}. \tag{19}$$

Using the parameters in Table 4, the Langmuir pressure is 1.5 MPa. The rebound pressure changes from 1.45 MPa at $P_{\mathrm{L}} = 1$ to 1.50 MPa at P_{L}=1.50 MPa and then decreases to -1.65 MPa with the Langmuir pressure increase to 9 MPa. The rebound pressure does not change monotonically with changes in P_{L}.

The higher the value of P_{L} is, the less the coal permeability decreases below the rebound pressure and the more the coal permeability increases above the rebound pressure.

Larger values of $\varepsilon_{\max}$ imply stronger swelling of coal, while smaller values of P_{L} indicate a lower pressure at which the expansion of coal reaches the same value. The larger $\varepsilon_{\max}$ and the smaller P_{L} are, the more the coal swells at the same pressure, and the more the permeability is affected by the expansion.

5 Conclusions

Coal permeability is an important parameter in methane control in mines and in CBM exploitation. The coal permeability is closely related to fractures and controlled by effective stress and matrix sorption deformation.

We developed a new coal permeability model under tri-axial stress conditions. In our model, the coal matrix is compressible, and Biot's coefficient varies between 0 and 1. The factor f_{m}, which is a parameter of the coal structure and is a constant between 0 and 1 for a specific coal, is introduced to measure the degree of influence of the coal matrix deformation on the fracture deformation. Matching the model to laboratory tests (Pini et al. 2009) showed that the factor f_{m} of the Sulcis coal sample for $N_2$and CO_2 is 0.1794 . The proposed permeability model is evaluated and compared to three of the most popular permeability models (the P–M model, S–D model and R–C model). The proposed model agrees well with the observed permeability changes and predicts the permeability of coal better than the other models.

The sensitivity of the proposed model to changes in the physical, mechanical and adsorption deformation parameters of the coal was investigated. Biot's coefficient and the bulk modulus affect the effective stress term in the proposed model, which in turn affect the permeability. The sorption deformation parameters and the factor f_{m} affect the coal matrix deformation term.

The effect of the coal parameters on permeability can be described using the rebound pressure, which is affected by such parameters as f_{m}, K, α, $\varepsilon_{\max}$ and P_{L}. The permeability decreases with increasing gas pressure at pressures below the rebound pressure and later increases gradually above the rebound pressure. The higher the parameters K, f_{m} and $\varepsilon_{\max}$ are and the smaller Biot's coefficient and Langmuir pressure P_{L} are, the more the coal permeability decreases below the rebound pressure, and the less the coal permeability increases above the rebound pressure.

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